Part III PAPER-V

Page No.:
Date: / /

Real Analysis Notes-5

Drichlet's test - If $b_n(x)$ is a monotonic function of n for each fixed value of x in $[a,b]$ and $b_n(x)$ tends uniformly to zero for $a \le x \le b$ and if there is a number $K > 0$, independent of x and n, such that for all values of x in $[a,b]$

$$\left| \sum_{r=1}^{n} u_r(x) \right| \le K, \forall n$$

then the series $\sum b_n(x)\, u_n(x)$ is uniformly convergent on $[a,b]$

First Method We may assume that $b_n(x)$ is a positive monotonic decreasing function of n, for each $x \in [a,b]$ since the general case follows the procedure given in the corollary of Abel's test.) Now $b_n(x)$ tends uniformly to zero, therefore for any $\epsilon > 0$ we can find an integer N (independent of x) such that for all values of

x in $[a, b]$

$$0 \leq b_n(x) < \epsilon / 2K \quad \forall \ n \geq N$$

For such values of n and any integer value of $p \geq 1$, we have, by Abel's Lemma:

$$\left| \sum_{r=n+1}^{n+p} b_r(x) U_r(x) \right| \leq b_{n+1}(x) \max_{q=1,2,\ldots p} \left| \sum_{r=n+1}^{n+q} U_r(x) \right|$$

$$\leq b_{n+1}(x) \left\{ \left| \sum_{r=1}^{n} U_r(x) \right| + \max_{q=1,2,\ldots n} \left| \sum_{r=n}^{n+q} U_r(x) \right| \right\}$$

$$< \frac{\epsilon}{2K} (K + K) = \epsilon$$

Hence by Cauchy's criterion the series $\sum b_n(x) U_n(x)$ converges uniformly for $x \in [a, b]$.